The saturation of spending diversity and the truth about Mr Brown and Mrs Jones

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Abstract

Several cross-country studies show that rising household income leads to consumption spending being spread more evenly across different spending categories (Clements et al., 2006). We argue that this result is likely due to aggregation. Using more disaggregated UK household spending data, we show that the spending diversity of households only rises up to a certain income level and then starts to decline as households concentrate more of their spending on particular expenditure categories that differ across households. It is precisely because of this growing heterogeneity on the household level that the average spending diversity of the population can nevertheless always rise in income. We build a model to capture this observed pattern and use it to show how the welfare judgements derived from studying a representative household become relatively more inaccurate when household income rises.

JEL classification: D12, C14, O33.

Keywords: Demand for variety, Engel’s Law, Spending Diversity.

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‘The preference hypothesis only acquires prima facie plausibility when it is applied to the statistical average. To assume that the representative consumer acts like an ideal consumer is a hypothesis worth testing; to assume that an actual person, the Mr. Brown or Mrs. Jones, who lives around the corner, does in fact act in such a way does not deserve a moment’s consideration.’ J.R. Hicks- A Revision of Demand Theory (1956) -

1 Introduction

One of the most salient features of modern economies is the wide range of goods and services available to consumers in markets. While much has been said about how firm behavior generates product variety, less has been said about how demand may also contribute to this phenomenon (Gronau and Hamermesh, 2008). Many models take the consumer demand for variety as given in the sense that the distribution of tastes in product space is fixed (Dixit and Stiglitz, 1977; Salop, 1977; Gronau and Hamermesh, 2008). Elsewhere, random utility models assume that the variation in consumer preferences is independent and identically distributed (McFadden, 1984; Calvet and Common, 2003). An implicit assumption in both cases is that the differences in tastes do not depend on economic factors such as the level of household income. Yet much evidence suggest that the contrary is the case: as people get rich, their demand for variety grows (Prais, 1952; Theil, 1967; Theil and Finke, 1983; Jackson, 1984; Falkinger and Zweimüller, 1996; Bils and Klenow, 2001; Bertola et al., 2014). The growth in the range of goods consumed is widely recognized to have vital implications for a range of economic issues: when the demand for different final goods changes with the level of income, this can lead to changes in the industrial composition and structural change (Pasinetti, 1981; Saviotti, 2001; Metcalfe et al., 2006; Foellmi and Zweimüller, 2008), impact the incentives to innovate (Foellmi and Zweimüller, 2006), as well as influence the realization of economies of scale (Bresnahan and Gambardella, 1998; Lipsey et al., 2008) and international trade flows (Hallak, 2010).

If variety demand does change with income, a major question is whether the direction of variety demand growth is similar or different across the population of consumers. In other words, as households get rich, do the types of new varieties they demand fundamentally differ across the population of consumers? Here previous studies suggest that the answer is negative as spending patterns are found to become more similar as income rises. Several studies have used entropy measures to calculate the distribution of spending across different expenditure categories, which we dub the ‘diversity of spending’ (Theil and Finke, 1983; Clements and Chen, 1996; Clements et al., 2006). These cross-country studies of spending patterns suggest that this diversity always increases when income rises. In other words, as their income grows, consumers appear to spread their
spending more evenly across all available goods and services. This ultimately suggests that household spending patterns will be distributed in a perfectly even fashion across all possible expenditure categories such that differences between consumer spending patterns will disappear as household income continues to rise.

We argue that this literature has ignored the possibility that heterogeneity in spending patterns are masked by high levels of aggregation present in cross-country data. Many recognize that it is crucial to study the precise relationship between aggregate and individuals behavior (Grandmont, 1987, 1992; Hildenbrand, 1994; Quah, 1997; Blundell and Stoker, 2005). A number of researchers have begun considering how behavioural heterogeneity can be modelled (Calvet and Common, 2003; Beckert and Blundell, 2008). This represents a departure from the main paradigm of postwar demand analysis that has concentrated on studying aggregates to verify representative agent models of behavior, even though these aggregates may not reflect actual household behavior. This paradigm is reflected in the above quote by J. R. Hicks, who argued that rather than attempting to account for actual household behavior, scholars should restrict their focus on average household behavior. It also underpins many commonly used models of demand analysis such as AIDS (Deaton and Muellbauer, 1980). However, if the observed level of heterogeneity in consumption patterns does grow with income, this traditional approach is increasingly untenable in middle and high income countries where heterogeneity in spending patterns are growing quickly.

In the case of spending diversity, whether to focus on average rather than actual behavior is particularly important. In this paper, we argue that as households shift their spending from basic necessities towards more discretionary categories, heterogeneity in spending patterns is likely to increase as household income rises and consumers concentrate their spending into different consumption areas. It is a well known fact that amongst the poorest, spending patterns are highly homogeneous across households as food spending tends to dominate household outlays (Banerjee and Duflo, 2007; Clements et al., 2006). The notion that heterogeneity of spending grows with income is consistent with Engel’s Law as well as evidence that Engel curves are highly heteroskedastic (Blundell and Stoker, 2005; Lewbel, 2008). Using UK household level spending data, we find evidence that suggest that diversity of household spending tends to fall at high income levels and that the overall differences in household spending patterns tend to grow (rather than fall). In other words, the truth about Mr. Brown and Mrs. Jones is that not only do they possess different spending patterns, but that these differences will grow as they become rich.

This emergent nature of consumption heterogeneity is worth taking into account. We develop a model that accounts for the fact that demand becomes more het-
heterogeneous when households get richer and that can explain why there can be a hump shaped relation between spending diversity and income at the individual level and a positive relation at the aggregate level. Here a key characteristic of the model is that differences between household spending patterns grow as household income increases. While there exist several models that consider how variety demand changes with income (Jackson, 1984; Theil and Finke, 1983; Gronau and Hamermesh, 2008) they do not tend to consider how variety demand may also grow in a heterogeneous manner across a population of consumers. Within this model setup, we analyze how much an increase in product variety is valued by individual households and by a representative household the preferences of which are such that the aggregate demand is the same for each good as in the case of consumer heterogeneity. When newly introduced goods have a high income elasticity (as typical for many innovations), we find that the representative household values an increase in product variety less (more) than individual households with heterogeneous tastes do when new goods are complementary (substitutable) to already consumed goods. Interestingly, these differences in welfare judgements between individual households and the representative household rise in the level of income. Estimating the gains from trade or innovation using representative agent models is therefore more likely to be misleading the richer the analyzed households are. As it is widely believed that the welfare effects of increasing product variety are substantial, this finding therefore calls for more sophisticated welfare analyses that take individual heterogeneity into account, in particular when rich economies are studied.

In terms of methodology, this paper studies the relationship between income and variety demand using cross sectional data. It may be tempting to study household spending patterns over time. However, the main obstacle in doing so is that one can not control for exogenous changes in variety demand over time. There has been rapid growth in the number of good available over time, which fundamentally affect the measurement of spending diversity. For this reason the main focus of this paper is on cross sectional results.

## 2 Stylized facts about spending diversity

We begin by comparing the manner in which the diversity of spending changes with income for individual households and representative consumers. We use annual household data sourced from the UK Family Expenditure Survey from 1990 to 2000. Over this time period the classification method for expenditure categories has been subject to change. To ensure consistency across sample period, the classification method specified by the Office of National Statistics in
2000 featuring $k = 12$ categories (see table in Appendix A) was selected and retrospectively applied to the data. We also exclude housing expenditure because of well-known problems with this data (Tanner, 1999; Blow et al., 2004). Savings was also excluded as it is non-consumption item. We have also censored data by removing Northern Ireland and households with more than two adults (which affects mainly share houses). We do, however, keep all households with two adults and any number of children. Regarding equivalence scales, OECD equivalence scales were used to control for differences in household composition.

For inflation, we have used the RPI Percentage change over 12 months - all items, excluding housing and mortgage payments (CDKG) produced by the UK Office of National statistics.

A problem that must also be faced when working with any household expenditure data is sample bias. As most household expenditure surveys have less observations at high levels of household income. However, in the case of the UK Family Expenditure Survey, Tanner (1999) studied the reliability of FES expenditure data by comparing it to spending figures found in the UK National Accounts. They found that the ratio of non-housing total FES expenditure to non-housing total expenditure in the National Accounts was around 90 per cent between 1974 and 1992. This instills us with some confidence that the magnitude of the sampling bias is not too large given that FES expenditure match the National accounts relatively well in this earlier period. Moreover, this problem is also mitigated by the fact that our sample sizes are relatively large.

We are concerned with the modeling the cross-sectional distribution of expenditure across expenditure categories, and the use of these models to shed light on various aspects of consumer behavior. There are $i \in [1, 2, ..., n]$ household with expenditure shares $j \in [1, 2, ..., k]$ such that $s_i = (s_{i1}, s_{i2}, ..., s_{ik})$ denotes the vector of expenditure shares across $k$ categories for household $i$. The total expenditure is denoted $x_i$ (also referred to as income) such that the overall expenditure on each item is $x_i \times s_{ij}$. Following (Theil, 1967; Theil and Finke, 1983; Clements et al., 2006), we do this by using an entropy measure of the expenditure shares:

$$E_i = -\sum_{j=1}^{k} \phi(s_{ij})$$

$$\begin{cases} 
\phi(s_{ij}) = s_{ij} \ln s_{ij} & s_{ij} > 0 \\
\phi(s_{ij}) = 0 & s_{ij} = 0 
\end{cases}$$

This quantity measures the degree of information (or ‘surprise’) one experiences when observing that a randomly selected dollar is spent on a specific item, subject to a logarithmic information function. For example if a household spends everything on food, it is known a priori that for that household any random dollar will be spent on this item, and thus the information received from this draw is zero. Conversely, if expenditure is equal over all categories, then we have no way of anticipating the category that a random dollar will be spent on, and
hence the information content is maximized.

In order to replicate the cross-country studies cited above, we order our samples of British households $x_1 < x_2 < \ldots x_n$ and partition them into decile subgroups. The expenditure shares are averaged within these subgroups to depict a representative household within that decile, and these are denoted $\hat{s}_{jd} = \frac{10}{n} \sum_{i \in d} s_{ij}$, where $d$ is the decile under consideration. The entropy of these shares $E_d(\hat{s}_{jd})$ is calculated alongside the average income level for households within that decile.

It should be noted that the first case corresponds with minimal diversity in expenditure (everything is spent on food) while the latter case describes perfect diversity. Thus $E$ takes on a value of zero when all the expenditure is concentrated in a single group, and is equal to $\ln(k)$ when the shares are exactly equal. As expenditure shares move away from the mean level $1/k$ the information content increases, and hence can be used as an indicator of diversity across the subgroups. The expenditure distribution within the highest decile is intuitively similar to the profile of an affluent country, while the distributions of middle and lower deciles are in some ways representative of poorer countries.

we calculate the of household spending household level and representative level households, and extract the underlying dependency with income with the use of Nadarya (1964) Watson (1964) kernel regressions. These are non-parametric regressions which are advantaged in that they do not assume a specific functional form for the relationship between $E$ and $x$. The estimator is:

$$
\hat{f}_h(x) = \frac{\sum_{i=1}^{n} K_h(x - x_i) E_i}{\sum_{i=1}^{n} K_h(x - x_i)}
$$

where $K$ is a Gaussian kernel and $h$ is the bandwidth which determines the degree of smoothness.

***FIGURE 1, 2 and 3 ABOUT HERE***

Figures 1 depicts spending diversity for the household level. The figure on the left depicts the case where goods are highly disaggregated (200+ expenditure categories). The figure on the right depicts the case where spending categories have been aggregates to 12 goods (See Table 1). In both cases there appears to be a visible inverted U-shape appearance of spending diversity across the income dimension. This contrasts strongly to results found on the more aggregated decile level shown in Figure 2. The Figure on the left-hand side reports spending diversity for representative household in each decile where goods and services are highly disaggregated (200+ expenditure categories). The Figure on the right-side show spending diversity has been aggregated into 12 categories (see Figure 1). In both cases the line is much flatter and there is little evidence that the diversity of spending fall at high income levels.
Beyond differences in the shape, there are also important differences in the levels of spending diversity between the household and the aggregate (decile) level. In Figure 3, the Figures on the left hand-side shows spending diversity of the decile consumer is situated above the average spending diversity estimated for the household level. This feature is consistent across 1990 (top), 1995 (middle) and 2000 (bottom). The Figures on the right-hand side show the calculated difference between decile level spending diversity and household level diversity in each year. Across all three years, the slope of the curve is positive, which suggests that these difference grow as household income rises. Note that at low income levels, the difference appears to be small. If there was no difference between the level of spending diversity on the individual and household level, then consumption patterns would be very homogeneous across the population of consumers. Indeed among the very poor, spending patterns are quite similar as most spending is dedicated to food (Banerjee and Duflo, 2007; Clements et al., 2006). However as household income rises, the differences appear to quickly grow, which implies that the heterogeneity in spending rises as households grow rich.

3 The model

This section develops a model that can account for the observed differences between household level and representative level spending diversity patterns. The focus on how spending diversity - i.e. how widely spending is dispersed across expenditure categories - was as developed by Theil (1967); Theil and Finke (1983); Theil and Clements (1987) and uniquely different from demand system analysis that focuses on how average demand for a system goods is dependent on prices, incomes, observable demographic characteristics and an error term that is assumed to be additive and posses a zero mean across the population of consumers. Instead of focusing on average demand, studies of spending diversity calculate how widely each household distributes its spending across different categories. When changes in this distribution are considered in relation to household income, it allows scholars to gain insights into the manner in which households expand the variety of goods consumed as their income rises. In this sense it is related to the ongoing issue of how to model zero expenditures in demand systems (Garcia and Labeaga, 1996; Fry et al., 2000).

While there do exist several models that consider how variety demand changes with income (Jackson, 1984; Theil and Finke, 1983; Gronau and Hamermesh, 2008) they do not tend to consider how variety demand may also grow in a heterogeneous manner across a population of consumers. Here a key characteristic
of the model below is that differences between household spending patterns grow as household income rises. Another set of models do consider heterogeneity by including a random component in the utility function which assumes differences in taste are stochastically distributed across the population of consumers (McFadden, 1984; Lewbel and Pendakur, 2009; Calvet and Common, 2003). In this regard, the novel aspect of this model is that income not only influence the variety demand of household, but it also influences the differences in household tastes. As such, differences in tastes across a population of consumers are not fixed, but rather tend to become more magnified by income growth.\footnote{Calvet and Common (2003) also analyze the relationship between demand heterogeneity and income. At the same time, strong assumptions are made that tastes are stochastically distributed across the population of consumers.}

The utility of individual $i$ is given by the generalized Stone Geary form

$$U_i = \left[ \sum_{j=1}^{k} \beta_{ij}^{2 \frac{1}{\varepsilon}} (q_{ij} - \gamma_j)^{\frac{1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}} \quad (2)$$

with $q_{ij}$ denoting the quality-adjusted quantity of expenditure item $j$ consumed by individual $i$ and $\gamma_j \geq 0$ the “subsistence consumption” level of item $j$. This utility function is only defined if $q_{ij} \geq \gamma_j$ holds, i.e. if the individual is rich enough to consume the subsistence level of all expenditure items with $\gamma_j > 0$ (when $\gamma_j < 0$, $q_{ij} > \gamma_j$ always holds). The parameter $\varepsilon > 0$ determines the degree of substitutability between expenditure items: in the limit case where $\varepsilon \to 0$, these items are perfectly complementary (utility is then given by $\lim_{\varepsilon \to 0} U_i = \min_{\gamma_j}(\beta_{ij}(q_{ij} - \gamma_j))$ while they become perfectly substitutable when $\varepsilon$ goes to infinity (when $\varepsilon = 1$ utility is given by the standard Stone-Geary form: $U_i = \prod_{j=1}^{k} (q_{ij} - \gamma_j)^{\beta_{ij}}$). The degree of substitutability therefore increases in $\varepsilon$.

It is assumed that $\sum_{j=1}^{k} \beta_{ij} = 1$ and that the parameters $\beta_{ij} \geq 0$ can vary across individuals while $\gamma_j$ is assumed to be the same for everyone. The subsistence consumption levels are therefore the same for all individuals (as they might reflect “biological” needs for food, shelter etc), while individuals might differ with respect to the relative importance that they attribute to consumption exceeding these levels.

Expenditures of individual $i$ are denoted by $x_i$ and the price of one unit of expenditure item $j$ by $p_{ij}$. The budget constraint of individual $i$ is therefore given by

$$x_i = \sum_{j=1}^{k} p_{ij} q_{ij} \quad (3)$$

Let us first consider the case where expenditures of individual $i$ lie above the
threshold \( \bar{x} \) which is required to purchase positive quantities of all goods \((q_{ij} > 0)\), i.e. in which \( x_i > \bar{x} \) (Condition A\(^3\)) holds. Setting up the Lagrangian \( L_i = U_i + \lambda_i \left[ x_i - \sum_{j=1}^{k} p_j q_{ij} \right] \) and deriving with respect to \( q_{ij} \) gives the first order conditions

\[
\frac{\partial L_i}{\partial q_{ij}} = U_i^{\frac{1}{\varepsilon}} \beta_{ij}^{\frac{1}{\varepsilon}} (q_{ij} - \gamma_j)^{-\frac{1}{\varepsilon}} - \lambda p_j = 0
\]

(4)

Dividing the first order conditions for items \( j \) and \( l \neq j \) by each other gives the condition

\[
\frac{q_{ij} - \gamma_j}{(q_{il} - \gamma_l)} = \frac{\beta_{ij}}{\beta_{il}} \left( \frac{p_l}{p_j} \right)^{\varepsilon}
\]

(5)

which can be rewritten as

\[
q_{il} = \gamma_l + \frac{\beta_{il}}{\beta_{ij}} \left( \frac{p_l}{p_j} \right)^{\varepsilon} (q_{ij} - \gamma_j)
\]

Solving the budget constraint (equation 3) for \( q_{ij} \) gives

\[
q_{ij} = \frac{x_i - \sum_{l \neq j} p_l q_{il}}{p_j}
\]

(6)

Inserting equation 5 into equation 6 allows to solve for the optimal quantity \( q_{ij}^* \) of expenditure item \( j \) by individual \( i \):

\[
q_{ij}^* = \frac{x_i - \sum_{l \neq j} p_l q_{il}}{p_j} \left( \frac{p_l}{p_j} \right)^{\varepsilon} \frac{\beta_{il}}{\beta_{ij}} \frac{\gamma_l}{\gamma_j}
\]

(7)

The optimal (quality-adjusted) quantity \( q_{ij}^* \) is a linear function of \( x_i \), implying linear Engel curves. As \( q_{ij} \) increases by \( \frac{1}{p_j + \sum_{l \neq j} \frac{p_l}{p_j} \frac{\delta \gamma_l}{\delta \gamma_j}} \) units for each unit that \( x_i \) increases, the slope of the Engel curve for item \( j \) increases in \( \beta_{ij} \) and decreases in \( p_j \). When individuals are poor (\( x_i \) is close to \( \bar{x} \)) they consume similar quantities of all items (\( q_{ij} \) is close to \( \gamma_j \) if \( \gamma_j > 0 \) or close to 0 if \( \gamma_j < 0 \)), while differences in \( \beta_{ij} \) across individuals can lead to large differences in the quantities of expenditure item \( j \) that they purchase when their income is large (relative to \( \bar{x} \)). Differences in the taste parameters \( \beta_{ij} \) across individuals can therefore generate the heteroskedasticity of Engel curves that is observed in the data (Prais and Houthakker, 1955; Härdle, 1990; Banks et al., 1997). The same pattern would also result if agents had the same preferences \((\beta_{ij} = \beta_j)\) but would face different prices \( p_{ij} \) for different expenditure items.

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\(^3\)In the case where \( \gamma_j \geq 0 \) \( \forall j \), Condition A is given by \( x_i > \sum_{j=1}^{k} p_j \gamma_j = \bar{x} \).
The income elasticity of demand for item \( j \) by individual \( i \) is given by
\[
\epsilon_{ij}(i) = \frac{\partial q^*_ij}{\partial x_i} \frac{x_i}{q^*_ij} = \frac{x_i}{x_i - \sum_{l=0}^{\infty} \frac{p_l \gamma_i}{\beta_{il}}} \frac{p_l}{p_i} \epsilon > 0
\] (8)
and therefore decreases if \( \gamma_j \) increases. The share of the budget that individual \( i \) spends on item \( j \) is given by \( s^*_ij = \frac{\epsilon_{ij} \beta_{il}}{x_i} \) and increases in \( \beta_{ij} \) as \( q^*_ij \) increases in \( \beta_{ij} \) if Condition A holds (as a strict inequality).

In order to show how the model can generate the pattern observed in the data, let us consider the following simple case: There are three expenditure items the prices of which are all equal to one \( (p_1 = p_2 = p_3 = 1) \) and two individuals with the same level of spending \( x_1 = x_2 = x \). There is a “basic need” item 1 with a positive subsistence consumption level \( \gamma_1 > 0 \), and two other items for which \( \gamma_2 = \gamma_3 = 0 \) (no subsistence consumption). It is assumed that \( \beta_{11} = \beta_{21} = \beta_1 \), \( \beta_{12} = \beta_{23} = 1 - \beta_1 \) and \( \beta_{13} = \beta_{22} = 0 \), implying that both individuals value additional units of item 1 in the same way, but that individual 1 values consuming item 2 but not item 3, while individual 2 values consuming item 3 but not item 2.

Using equation 7, the optimal consumption levels can be derived as \( q^*_{11} = q^*_{21} = \beta_1 x + \gamma_1 (1 - \beta_1) \), \( q^*_{12} = q^*_{23} = (1 - \beta_1) (x - \gamma_1) \), and \( q^*_{13} = q^*_{22} = 0 \), implying the following optimal budget shares:
\[
s^*_{11} = s^*_{21} = \beta_1 + \frac{\gamma_1 (1 - \beta_1)}{x}
\] (9)
\[
s^*_{12} = s^*_{23} = (1 - \beta_1) \frac{1 - \gamma_1}{x}
\] (10)
\[
s^*_{13} = s^*_{22} = 0
\] (11)

When spending is at its minimal level \( x = \frac{x}{x} = \gamma_1 \), individuals spend all money on item 1, implying that \( s^*_{11} = s^*_{21} = 1 \) and \( s^*_{12} = s^*_{23} = 1 \). When \( x \) is raised above the level \( \frac{x}{x} \), \( s^*_{11} \) and \( s^*_{21} \) fall, while \( s^*_{12} \) and \( s^*_{23} \) rise (and approach the value \( 1 - \beta_1 \) when \( x \) goes to infinity).

The levels of spending diversity in consumption spending are given by \( E^*_1 = -s^*_{11} \ln s^*_{11} - s^*_{12} \ln s^*_{12} - s^*_{21} \ln s^*_{21} - s^*_{23} \ln s^*_{23} \) (note that \( 0 \ln 0 = 0 \)). When spending is at the minimal level \( x = \frac{x}{x} = \gamma_1 \), spending diversity is zero \( (E^*_1 = 0) \), while it reaches the value \( \lim_{x \to \infty} E^*_1 = -\beta_1 \ln \beta_1 - (1 - \beta_1) \ln (1 - \beta_1) > 0 \) when \( x \) goes to infinity. Given that \( \beta_{12} = \beta_{23} = 1 - \beta_1 > \frac{1}{2} \), \( E^*_1 \) first rises and then falls
in $x$ when $x > \bar{x}$ (see Figure 5). This is because households spend most of their money on the basic need item 1 when they are poor and (given that $1 - \beta_1 > \frac{1}{2}$) most of it on either item 2 or 3 when they are rich, implying that the level of spending diversity is largest for an intermediate value of $x$ (for which expenditures are evenly distributed across items$^4$). As seen in the data (see Figure 1 and 2), there is therefore an inverted-U shaped relationship between spending diversity and income.

Let us now consider the spending diversity of aggregate consumption spending. As both individuals spend the same share of their income on item 1, the aggregate spending share $\hat{s}_1$ of item 1 is equal to the individual shares, i.e.

$$\hat{s}_1 = s_{11} = s_{21}$$

As aggregate expenditures are equal to $2x$ and as only individual 1 purchases item 2 and only individual 2 purchases item 3, the aggregate spending shares on these items are given by

$$\hat{s}_2 = \hat{s}_3 = \frac{s_{12}}{2} = \frac{s_{23}}{2} = (1 - \beta_1) \frac{1}{2} - \frac{\gamma_1}{2x}$$

Using this equation, the spending diversity of aggregate expenditures can be derived as

$$\dot{E} = -\hat{s}_1 \ln \hat{s}_1 - \hat{s}_2 \ln \hat{s}_2 - \hat{s}_3 \ln \hat{s}_3 = -s_1 \ln s_1 - 2 \frac{s_{12}}{2} \ln \frac{s_{12}}{2} = E_1 + s_{12} \ln 2$$

As $s_{12} \ln 2 = (1 - \beta_1) \left(1 - \frac{3}{2}\right) \ln 2$ is equal to zero for $x = \bar{x}$ and continuously rises in $x$ when $x > \bar{x}$, the spending diversity of the aggregate expenditures always exceeds that of individual expenditures, and the more so, the larger $x$ is (see Figure 4). Moreover, the spending diversity $\dot{E}$ of aggregate expenditures either continuously rises in $x$ or reaches a maximum at a level of $x$ that is larger than the one for which the spending diversity of individual consumption is maximal$^5$.

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$^4$Inserting $s_{12} = 1 - s_{11}$ into $E = -s_{11} \ln (s_{11}) - s_{22} \ln (s_{22})$ and deriving with respect to $s_{11}$ gives the first order condition $\ln (s_{11}) = \ln (1 - s_{11})$ which implies that spending diversity is maximal if $s_{11} = \frac{1}{2}$ (the second derivative is always negative so that this is a global maximum).

$^5$Taking into account that $\hat{s}_2 = \hat{s}_3 = \frac{1 - \beta_1}{x}$, we can write $E = -\hat{s}_1 \ln \hat{s}_1 - (1 - \hat{s}_1) \ln \left(\frac{1 - \hat{s}_1}{x}\right)$. Deriving with respect to $\hat{s}_1$ gives the first order condition $\ln (1 - \hat{s}_1) - \ln \hat{s}_1 = 2 + \ln 2$ which must be satisfied when $\dot{E}$ is maximal (the second derivative is negative). As the left hand side of this first order condition continuously falls from plus to minus infinity when $\hat{s}_1$ goes from 0 to 1 and is equal to zero at $\hat{s}_1 = \frac{1}{2}$, the first order condition is satisfied for some interior value $\hat{s}_1$, with $0 < \hat{s}_1 < \frac{1}{2}$. As $\hat{s}_1 = s_1 > \beta_1$ (note that $\lim_{x \to \infty} \hat{s}_1 = \beta_1$), $\dot{E}$ therefore continuously rises in $x$ if $\hat{s}_1 < \beta_1 < \frac{1}{2}$, and reaches a maximum for a finite level of expenditures if $\beta_1 < \hat{s}_1$. In the latter case, the maximum of the hump is situated at a larger level of $x$ than the maximum of the hump of individual spending diversity as $E$ is maximal if $\hat{s}_1 = \frac{1}{2}$, while $\dot{E}$ is maximal if $\hat{s}_1 = s_1 < \frac{1}{2}$.
Those are exactly the pattern found in the data (see Figures 1 and 2).

The main point of the model is that the heterogeneity of demand is emergent in the sense that differences in consumer spending patterns grow as household income rises. While at low income levels, demand is relatively homogeneous, differences in spending patterns become more distinct at high income levels. This accounts for why increasing diversity on the aggregate level is consistent with declining spending diversity on the household level.

4 The value of product variety

In order to design appropriate innovation, trade, and antitrust policies it is crucial to be able to evaluate how much individual households value an increase in product variety. In the following, the model from above is used in order to study the welfare implications of increasing the variety of consumption items available to consumers.

It is assumed that initially only item 1 exists and that items 2 and 3 can be introduced through innovation or can be made available through a free trade agreement. While item 1 is sold at price $p_1 = 1$, items 2 and 3 (when available) are both sold at price $p_2 = p$ (and at an infinite price when not available). In order to allow to compare the welfare levels with and without items 2 and 3, it is assumed that $\gamma_2 \leq 0$ and that $\gamma_3 \leq 0$ hold, i.e. that there is no required subsistence consumption level for these goods. It is assumed that $\gamma_2 = \gamma_3$ holds. While $\beta_{i1}$ (the welfare weight on item 1) is assumed to be the same for all individuals and equal to the constant $\beta_{i1} = 1 - \beta$, the degree to which individual $i$ prefers item 2 over item 3 is allowed to vary within the range where the individual still purchases positive quantities of all available items and in which $\beta_{i2} + \beta_{i3} = \beta$ holds.

From equation 7 we can infer that $q_{i1}$ and also the sum $q_{i2} + q_{i3}$ only depend on the aggregate welfare weight $\beta$ for items 2 and 3, but not on how much items 2 and 3 are liked by a particular individual. When there are two households with the same level of spending for which the weights on items 2 and 3 are reversed (i.e. $\beta_{12} = \beta_{23}$ and $\beta_{13} = \beta_{22}$) the aggregate demand $q_{ij}^* = q_{ij1} + q_{ij2}$ for items $j = 1$, $j = 2$ and $j = 3$ therefore only depends on $\beta$ and not on the individual values $\beta_{ij2}$ and $\beta_{ij3}$. Aggregate demand is therefore the same as in the case where both households value both items equally ($\beta_{ij2} = \beta_{ij3} = \frac{\beta}{2}$) and can also be derived from the utility maximization problem of a representative individual $i = r$ with preference parameters $\beta_{r1} = 1 - \beta$ and $\beta_{r2} = \beta$, $\beta_{r3} = \frac{\beta}{2}$ and expenditures $x_r = 2x$.
with heterogeneous tastes value an increase in product variety in a different way than a representative household that demands the same aggregate quantities of all items.

While it is obvious that households benefit more from the introduction of a good that they like a lot than from the introduction of a good that they do not like, the question considered here is whether a household benefits more or less from the joint introduction of both item 2 and 3 when it puts a larger or a lower relative welfare weight on one of them keeping $\beta_2 + \beta_3 = \bar{\beta}$ and therefore the total quantity of the two goods that it consumes constant\(^6\). To which extent a household values variety is measured by the amount $F_i$ of income $x_i$ (or by the quantity $F_i$ of item 1 if only this item is sold at $p_1 = 1$) that it is maximally willing to give up in order to be able to also purchase items 2 and 3 at price $p$ (with the remaining income $x_i - F_i$).

**Proposition 1.** a) Suppose that $\epsilon = 1$ and $\gamma_2 = \gamma_3 = 0$ hold (homothetic preferences for items 2 and 3). The amount of income $F_i$ that household $i$ is maximally willing to give up in order to be able to not only consume item 1, but also items 2 and 3 is then independent of the relative welfare weights $\beta_2$ and $\beta_3$ as long as $\beta_2 + \beta_3 = \bar{\beta}$ holds. The household then values the increase in product variety in the same way as a representative household with the same income and with the “average” preferences $\beta_{r2} = \beta_{r3} = \frac{\bar{\beta}}{2}$ (that generate the same aggregate demand).

b) Suppose that $\gamma_2 = \gamma_3 < 0$ holds (non-homothetic preferences for items 2 and 3). When $\epsilon < 1$ ($\epsilon > 1$), $F_i$ increases (decreases) in $\beta_{ij}$ if $\beta_{ij} > \frac{\bar{\beta}}{2}$ holds. When $\epsilon < 1$ ($\epsilon > 1$) holds, each household then values variety more (less) than the representative household and the more (less) so, the more heterogeneous individual tastes are, i.e. the larger $\beta_{ij}$ is. When household income $x_i$ increases, the differences in welfare judgements between individual households and the representative household increase: When $\epsilon < 1$ ($\epsilon > 1$) holds, raising $\beta_{ij}$ above the level $\bar{\beta}_2$ increases (decreases) $\frac{E_i-p_i}{\beta_{ij}}$ more, the larger $x_i$ is.

**Proof.** See Appendix A

Even though aggregate consumption can be derived from the utility maximization problem of a representative household with average preferences, this household might value an increase in product variety in a different way than individuals do: While welfare judgements turn out to be the same when preferences are homothetic, studying the welfare of a representative household without taking the empirically observed heterogeneity into account leads to distorted results when the newly introduced goods have a high income elasticity (i.e. negative

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\(\text{\textsuperscript{6}}\)By looking at the joint introduction of two goods, one does not need to consider individual risk preferences that might play a role when instead the welfare consequences of the introduction of only one item of ex ante unknown desirability were studied.
values of $\gamma_j$, which seems to be a prevalent feature of newly invented goods). In this case, the representative household values an increase in product variety less than individual households with heterogeneous tastes do when goods are complementary ($\epsilon < 1$), but values it more when goods are substitutable ($\epsilon > 1$). Interestingly, these differences in welfare judgements rise in the level of income. This implies that the representative agent model delivers more accurate welfare results when households are poor and concentrate their expenditures in similar ways on basic need goods and less accurate results when households are richer and start concentrating their expenditures on certain luxurious goods. While the representative agent model might still deliver sufficiently accurate welfare predictions when poor economies are studied, it might therefore fail to do so when households become richer and when individual tastes become the dominating factor of consumption pattern. While the representative agent framework was probably a useful paradigm in the times of Hicks, the current analysis suggests that its usefulness might have declined due to economic growth.

4.1 Accounting for variety demand

In the above sections, we only considered the case in which each household purchases non-negative quantities of all items. The model can, however, also account for the empirically observed fact that richer households demand a larger variety of items: when $\gamma_j < 0$, only households with sufficiently large income purchase a positive quantity of item $j$. While items for which $\gamma_j > 0$ are purchased by all individuals, the marginal utility of the first unit of an item with $\gamma_j < 0$ might be so low that only rich individuals purchase it. When there are many items for which $\gamma_j < 0$ holds, the model can therefore generate the pattern that richer households demand a larger variety of items.

In the following, the simple example from above is analyzed in order to show how preference heterogeneity can affect the way in which households increase their variety demand as their income grows:

Suppose again that the prices of all items are equal to zero and that there is a “basic need” item 1 with a positive subsistence consumption level $\gamma_1 > 0$, and two more luxurious items for which $\gamma_2 = \gamma_3 < 0$ holds. There are again two households that value additional units of item 1 in the same way, but that have opposing tastes with regard to items 2 and 3 as $\beta_{11} = \beta_{21} = \beta_1$, $\beta_{12} = \beta_{23} = 1 - \beta_1$ and $\beta_{13} = \beta_{22} = 0$ holds (individual 1 values consuming item 2 but not item 3, while individual 2 values consuming item 3 but not item 2). Comparing the utility derived from only consuming item 1 to that derived from consuming item 1 in combination with the preferred of the other two items, it can be shown that
households prefer to only consume item 1 as long as the following condition holds:

$$x \leq \gamma_1 - \gamma_2 \beta_1 \frac{\beta_1}{1 - \beta_1}$$

Household 1 therefore only starts consuming item 2 and Household 2 only starts consuming item 3 when their incomes raise above the level

$$x = \hat{x} \equiv \gamma_1 - \gamma_2 \frac{\beta_1}{1 - \beta_1}$$

(that positively depends on $\gamma_1$ and $\beta_1$). The two types of households therefore expand their variety demand into different directions.

Applying these insights to a more general setting with many items that can be grouped into broader consumption categories (each comprising items that are similar with respect to the parameter $\gamma_j$), the direction in which variety demand grows varies across the population. This implies that the diversity of the variety demand of a household with income $x$ is lower than the diversity of the average consumption basket of different households with income $x$.

This is indeed the case when we look at the data: Figure 5 presents the spread of variety demand across 12 expenditure categories at the household level using data from the year 2000. Specifically, the number of varieties consumed in each expenditure category was counted and the overall fraction of varieties consumed in each of the 12 expenditure categories was calculated. The entropy measure described in Section 2 was then applied to these fractions in order to estimate the dispersion of household variety demand across the 12 expenditure categories. Figure 6 presents the diversity of variety demand on the representative (decile) level for the same year. Figure 7 presents both the household and decile levels together. These graphs show that the diversity of variety demand at the household level is lower than the diversity of variety demand at the representative (decile) level and that the diversity of variety spending increases in household income. As different households grow their variety demand, the consumption baskets therefore become more diverse in terms of variety consumed across different expenditure categories.

## 5 Conclusion

The truth about Mr Brown and Mrs Jones is that not only do they possess different spending patterns, but that these differences tend to grow with income levels. In this paper we have highlighted how this ‘emergent’ aspect of consumption heterogeneity has important implications for the extent to which the behavior of the representative consumer reflects the actual behavior and preferences of individual consumers.

While at the aggregate level the spread of household expenditure across cate-
gories, i.e. the diversity of spending, continues to rise as income grows, this not the case when examining the diversity of expenditures at the household level. Rather, household spending patterns on the more disaggregated level show that rich households tend to concentrate their spending patterns into particular expenditure categories. Because each household concentrates into different types of expenditure categories, diversity of household expenditure can nevertheless increase at the aggregate level while it declines at the individual level.

These findings, in combination with the results obtained in the theoretical analysis, highlight the growing pitfalls of adopting representative agent models in a world where rising income is driving ever increasing heterogeneity in consumption patterns. Paying attention to what Mr Brown and Mrs Jones do instead of only focusing on average behavior should therefore become a priority for future research.

6 Appendix A

Proof. Let us define \( x_i \equiv \bar{x}_i + F_i \). Individual \( i \) must be indifferent between only consuming item 1 and having income/spending \( x_i \) and consuming all three items and having income/spending \( x_i - F_i = \bar{x}_i \). Using equation 2, this implies the following equation:

\[
1 - \frac{1}{\bar{x}^2_1}\frac{1}{(\bar{x}_i + F_i - \gamma_1)^{\frac{1}{\bar{x}_1}}} + \beta_{i2}^2 (-\gamma_2)^\frac{1}{\bar{x}_2} + \beta - \beta_{i2}^2 \frac{1}{(\gamma_2)^\frac{1}{\bar{x}_2}} = 1 - \frac{1}{\bar{x}_1^2}\frac{1}{(q_1(\bar{x}_i) - \gamma_1)^\frac{1}{\bar{x}_1}} + \beta_{i2}^2 (q_2(\bar{x}_i) - \gamma_2)^\frac{1}{\bar{x}_2} + \beta - \beta_{i2}^2 (q_3(\bar{x}_i) - \gamma_2)^\frac{1}{\bar{x}_2}
\]

Subtracting the right hand side (RHS) from the left hand side (LHS) and defining \( Q \equiv LHS - RHS \), we can implicitly differentiate this equation and obtain \( \frac{dF_i}{d\gamma_2} = -\frac{\partial Q}{\partial \gamma_2} \). We therefore analyze how \( F_i \) depends on \( \beta_{i2} \), taking \( \bar{x}_i \) as given (and \( x_i \) to be variable), as this simplifies the analysis. This yields the same qualitative results as studying how \( F_i \) depends on \( \beta_{i2} \), taking \( x_i \) as given. We obtain \( \frac{\partial Q}{\partial \gamma_2} = \frac{1}{\bar{x}_1}(1-\beta)^{\frac{1}{\bar{x}_1}}(\bar{x}_i + F_i - \gamma_1)^{-\frac{1}{\bar{x}_1}} \). Moreover, \( \frac{\partial Q}{\partial \gamma_2} = \frac{1}{\bar{x}_1}(\bar{x}_i + F_i - \gamma_1)^{-\frac{1}{\bar{x}_1}} (\beta_{i2})^{\frac{1}{\bar{x}_2}} - \beta - \beta_{i2}^2 \frac{1}{\bar{x}_1} \)

holds (in order to show this, the condition \( \frac{q_2(q_1 - \gamma)}{\beta - \gamma} = q_2(q_1 - \gamma) \), which can be derived from the consumers first order conditions, was used).

When \( \gamma_2 = \gamma_3 = 0 \) or when \( \beta_{i2} = \frac{\beta}{\bar{x}_2} \), we obtain \( \frac{\partial Q}{\partial \gamma_2} = 0 \). Given that \( \epsilon = 1 \), this implies that \( \frac{dF_i}{d\gamma_2} = 0 \) and that (by symmetry) \( \frac{dF_i}{d\gamma_3} = 0 \) must hold. When \( \beta_{i2} > \frac{\beta}{\bar{x}_2} \), \( \text{sign} \frac{\partial Q}{\partial \gamma_2} > 0 \) holds, implying that \( \text{sign} \frac{dF_i}{d\gamma_2} = \text{sign}(1-\epsilon) \) when \( \epsilon = 1 \). Due to symmetry, also \( \text{sign} \frac{dF_i}{d\gamma_3} = \text{sign}(1-\epsilon) \) holds if \( \beta_{i2} > \frac{\beta}{\bar{x}_2} \).

Differentiating \( \frac{dF_i}{d\gamma_2} \) with respect to \( \bar{x}_i \) gives \( \text{sign} \frac{\partial dF_i}{\partial \bar{x}_i} = \text{sign} \frac{\partial Q}{\partial \gamma_2} \frac{\partial Q}{\partial F_i \partial \gamma_2} = \text{sign} \frac{\partial dF_i}{d\gamma_2} \). As \( \text{sign} \frac{\partial Q}{\partial F_i \partial \gamma_2} = \text{sign}(1-\epsilon) \) and as \( \frac{\partial Q}{\partial \gamma_2} > 0 \) when \( \beta_{i2} > \frac{\beta}{\bar{x}_2} \) and
\( \gamma_2 < 0 \) hold, we therefore get \( \text{sign} \frac{\partial \frac{d\gamma_2}{dx_i}}{\partial \beta_2} = \text{sign}(1 - \epsilon) \) under these conditions, implying that an increase in \( x_i \) makes \( \frac{d\gamma_2}{dx_i} \) more positive if \( \epsilon < 1 \) holds and more negative if \( \epsilon > 1 \) holds. When \( \beta_{2i} = \frac{\bar{\beta}}{2} \), \( F_i = F_r \) and household \( i \) values variety in the same way as the representative household. When \( \epsilon < 1 \) (\( \epsilon > 1 \)), the condition \( \text{sign} \frac{\partial \frac{d\gamma_2}{dx_i}}{\partial \beta_2} = \text{sign}(1 - \epsilon) \) therefore implies that raising \( \beta_{2i} \) above the level \( \frac{\bar{\beta}}{2} \) increases (decreases) \( \frac{F_i - F_r}{x_i} \) more, the larger \( x_i \) is (note that \( F_i < F_r \) when \( \epsilon > 1 \)).
References


7 Figures

Figure 1: Spending Diversity at the household level

Notes: The Figure on the left shows household level spending diversity when spending is disaggregated across 200+ categories. The figure on the right shows household level spending diversity when 12 aggregate categories of household expenditure are used (see Table 1). In both cases, there is an inverted U-shape relationship between spending diversity and income. The number of observations was 6,047 in 1990, 5,984 in 1995 and 5,865 in 2000.

Figure 2: Spending diversity at the representative (decile) level

Notes: The Figure on the left shows decile level spending diversity when spending is disaggregated across 200+ categories. The figure on the right shows decile level spending diversity when 12 aggregate categories of household expenditure are used (see Table 1). In both cases, decile level spending diversity appears to grow as household income increases.

Figure 4: spending diversity of aggregate expenditure versus individual expenditure
Figure 3: Difference between household and aggregate levels of spending diversity

Notes: The top row shows data from 1990, the middle row shows data from 1995 and the bottom row shows data from 2000. The figures on the left-hand-side depict spending diversity on the household level (solid line) and on the decile level (dashed line). The dots show the individual observation for household level spending diversity. The right-side plots the difference between decile level spending diversity and household level spending diversity and income. This shows that differences between the household level and the decile level tends to grow as household income rises.
Figure 5: Diversity of variety demand on the household level (2000)

Note: This figure reports how evenly the varieties consumed by a household are distributed across the 12 categories (see Table 1). This figure shows that initially this diversity increases and then flattens out.

Figure 6: Diversity of variety demand on the decile level (2000)

Note: This figure reports how evenly the varieties consumed by a representative household (decile level) are distributed across the 12 categories (see Table 1).
<table>
<thead>
<tr>
<th>Category</th>
<th>Examples of spending</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>Milk, Eggs, vegetables, meats, sweets, non-alcoholic beverages. Take away meals, food bought and consumed at work and school.</td>
</tr>
<tr>
<td>Fuel Light and Power</td>
<td>Gas, Electricity, Coal, bottled gas, paraffin, wood.</td>
</tr>
<tr>
<td>Alcoholic Drinks</td>
<td>Beer, Lager, Cider, Spirits Liqueurs.</td>
</tr>
<tr>
<td>Tobacco</td>
<td>Cigarettes, Pipe tobacco, cigars</td>
</tr>
<tr>
<td>Clothing and Footwear</td>
<td>Outerwear, Underwear, Clothing accessories, Footwear, Haberdashery and clothing materials</td>
</tr>
<tr>
<td>Household goods</td>
<td>Furniture and Furnishings, Electrical and gas appliances. Toilet paper, Pet and garden expenditure.</td>
</tr>
<tr>
<td>Domestic and Paid services</td>
<td>Childcare, domestic help, laundry, postage and telephones, subscriptions and stamp duty.</td>
</tr>
<tr>
<td>Travel</td>
<td>Fares, other transport costs, Purchase and maintenance of non-motor vehicles.</td>
</tr>
<tr>
<td>Entertainment and Education Services</td>
<td>Cinema, spectator sports, TV rental and subscription, hotels and holiday expenses, betting stakes, educational fees and maintenance, Ad hoc school expenditure, betting stakes.</td>
</tr>
</tbody>
</table>